

The Force and Gravity of Events

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Local events are characterized by where, when and what. Just as (bosonic) spacetime forms the backdrop for location and time, (fermionic) property space can serve as the backdrop for the attributes of a system. With such a scenario I shall describe a scheme that is capable of unifying gravitation and the other forces of nature. The generalized metric contains the curvature of spacetime and property separately, with the gauge fields linking the bosonic and fermionic arenas. The super-Ricci scalar can then automatically yield the spacetime Lagrangian of gravitation and the standard model (plus a cosmological constant) upon integration over property coordinates.

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1. Life with Salam: Personal Reminiscences

This conference commemorates the life and achievements of Abdus Salam. It is therefore incumbent upon me to begin by drawing a picture of the man with a few private reminiscences that may convey something about his greatness, genius and humanity. I know that *some* of you have worked with him in *some* capacity at *some* stage, but I suspect that only very few of you will have had the privilege of interacting 17 years with Abdus Salam as I have done: first as an undergraduate, then as a postgraduate, postdoc, and eventually as an academic colleague and scientific collaborator. I shall cover the period 1959-1976 when I was closely involved with him; there are a few others present here who can competently fill in the later years to leave you with a more complete portrait of Salam. If certain members of this audience have heard my vignettes of him before, I apologize in advance, but with these reminiscences you may at least enjoy reliving fond memories of him.

Salam was a man in a hurry; his reputation preceded him everywhere. As a lowly undergraduate student I first came across him in 1959 when we had to choose our third year specialty by making a selective tour of the various research departments at Imperial College. At that time Salam was housed in the Mathematics section (before moving to Physics in 1960) and I can vividly remember his verve and vivacity as he explained to us his latest pet project, which happened to be chiral symmetry and gamma-5 invariance for favouring massless left-handed neutrinos and leading to parity violation. That discourse went right over all our heads at the time but they coaxed me at least to turn to theoretical physics for my final year specialty. I am sure others have succumbed to Salam's persuasive abilities on whatever topic he expounded. That very year he taught us advanced quantum mechanics a-la-Dirac, whom he idolized; his lectures seemed pretty good to me, so in 1960 I embarked

on my PhD, with Salam acting as my supervisor from 1961 onwards. During those years Salam's areas of interest were on vanishing of renormalization constants for composite systems, Lie Groups and on the "Gauge Technique". He took a keen interest in my research topic and would enquire every morning as to what progress I had made – putting great pressure on me, as he did on all his other students. He was always bubbling with new ideas and postgraduates found it very hard to survive his changes of tack or emphasis; but it definitely steeled us.

That period saw the development of the eightfold way and I very well remember Salam's heated arguments with Neeman that emanated from his office across the corridor. Salam lost out on the birth of $SU(3)$ because of his insistence on a fundamental (Sakata) triplet so you can say that he tripped up on that. However in hindsight was he that far out? Think quarks and you will agree that his intuition was amazing. Yes, he could make mistakes — and who does not — but on most things his inspiration was spot on. When I sometimes asked him where and how he got his latest idea, he would give a wicked smile and point upwards. He always moved on to something new when an old idea was established and played out; he was never one for pot-boilers and he never suffered fools gladly, in private anyway. When he became somewhat contemptuous of the work of some scientists he referred to them as 'tom-tits' or 'broken reeds' or 'youths'. However in public he was always polite and he encouraged anyone who presented a new concept. If there is one lesson that Salam has taught me it is that one should not be ashamed to move on if a concept is not bearing fruit. That may explain why there was always great anticipation whenever he delivered a lecture on some topic: the expectation was that he would spring something new on the audience.

Salam was a demon for hard work. For instance, in the summer of 1967 he had an appendectomy; I visited him in hospital two days after his operation and it was not long before he launched into discussing multi-quark states and their current algebras, despite his obvious physical discomfort. He was well travelled and especially during 1962-65 when in the process of setting up ICTP; funding problems beset him for a good while and he would rile at politicians who opposed his initiative, including the Australian representatives in UNESCO! Owing to his regrettable experiences in Pakistan after leaving Cambridge and his constant bemoaning of the decline of Islamic science, he felt a driving need to found such a centre to assist third world countries. He had a special affinity for isolated scientific personnel who, like himself at first, struggled to keep abreast of the latest advances. He believed that initially concentrating on theoretical studies would serve the purpose as it would cost relatively little but represented the forefront of physics; later on the Centre could be used as a launchpad for other branches of science. At Imperial College, and later at Trieste, Salam became a major magnet for Pakistani students as well as those from Africa and Latin America. The place was abuzz with them and Salam took great pains to foster their work. His initiatives and his constant movement gave him little time for relaxation and I vividly remember several meetings that

John Strathdee and I had with him in the hotel lobby at Trieste, after his energy-sapping perambulations. I think he was able to maintain his stamina because he was an early riser and went to bed early too. My request that he not ring me before 7am, unless there was an emergency, must have tested his self-control.

Always the perfect host, he warmly welcomed new visitors to IC and ICTP by inviting them to dinner at his home or elsewhere. He was not in the least pretentious about the venue. I recall one occasion at IC when Bruno Zumino came to give us a lecture. Instead of taking him to the Staff Club for lunch he opted for the College cafeteria so he could mingle with the ‘plebs’ and sample the canteen fare, which he rather savoured! The thing that most impressed John and I about his eating habits was when we were consuming fish; Salam would crunch his way through the spine and bones, leaving only the head and tail! If waiters were tardy with producing the bill, Salam would get up and leave the ‘trattoria’ when his patience ran out; the sight of the ‘camerieri’ scurrying after him with ‘il conto’ was pure comedy. At coffee he would often ruminate about the heyday of Arabic science and how vital it was in the Middle Ages for passing on the Greek scientific legacy to Europe via Spain.

I have been asked by the organizers to comment upon the the birth of the standard model during 1967 and Salam’s prominent role in it. This is an excellent occasion to set the record straight and recount my view of its history; if nothing else to refute innuendos which have occasionally surfaced during the 1970s that Salam was not deserving of the Nobel Prize. That autumn of 1967 I had been in charge of organizing the seminars at IC. Because Salam was constantly on the move and hardly spent more than one month at a stretch in London, I arranged with him to give a couple of lectures on his recent research (in October, to the best of my recollection) during his spell at IC to kick off the seminar season, as it was early in the academic year. He agreed to do so even though the audience attending those talks was somewhat thin. Paul Matthews was certainly present, but Tom Kibble was away in sabbatical in the USA. My memory of his lectures is a bit indistinct nowadays, but I do remember that he kept on invoking these k -meson tadpoles which disappeared into the vacuum which induced the spontaneous breaking of the gauge symmetry: what we now know as the expectation value of the Higgs boson. The resulting model looked rather ugly – and it still is – and I admit that I paid little attention to it; nor do I think that Salam himself was especially enraptured by the model’s beauty. A week or so later, I wandered into the Physics Library and came across Steven Weinberg’s Physical Review Letter, which I noticed looked suspiciously like Salam’s attempt. I showed the article to Salam, who was rather troubled that it was almost the same as his own research, but which was of course entirely *independent*. Matthews and I urged him to publish his work at the earliest opportunity and this happened to be the upcoming Nobel Symposium. As they say, “the rest is history”. I hope that this account of the events at the time scotches all aspersions that Salam should not have been a prize recipient.

Above all Salam impressed upon us the importance of tackling challenging problems: to prospect for new scientific fields and abandon raking over old coals, if one was to make one's mark.. He lived up to that precept throughout his life, in spite of accusations that he had a scattergun approach to physics. It is a lesson that some young scientists today should heed. In his last years, despite his grave illness, he addressed the puzzle as to why certain life forms have particular handedness; what could be more fundamental or significant than that? I know that I miss his wise words, his friendship, his guidance, his generosity and his humanity. This conference is therefore a personal acknowledgement of how much he helped to shape my own career. More widely, it is a timely reminder of how much the scientific landscape and international developments owe to him. The ICTP is a permanent testament to that.

2. An algebraic framework for events

This brings me to the scientific part of my talk. The material which I will present is sufficiently different from other attempts at unification of forces that I rather fancy Salam might have given it a nod of approval. Two years ago, at the Dyson 90th anniversary conference, I outlined¹ how it is possible to unify gravity with the simplest of all forces, electromagnetism – Einstein's eldorado – simply by appending a single complex anti-commuting Lorentz *scalar* variable to spacetime, *not a spinor*; importantly no infinite KK modes arise. My partner in crime (Paul Stack) and I have made considerable progress since then and I will now try to summarise how to unify gravity with the other forces of nature through a relatively simple supermetric. Our attempts in this direction have been motivated by the present parlous state of particle physics and the snail's pace of progress in this area over the last 40 years. Here is a statement which may bring me some opprobrium: namely, apart from the timely discovery of the Higgs boson, emergence of multi-quark states and significant astrophysical advances, there is very little to celebrate in our attempts to unravel nature at the most basic level. This is in spite of determined, quasi-herculean efforts of theorists who have persistently espoused/promoted very clever ideas. So far, Nature stubbornly refuses to cooperate by providing us with unequivocal experimental signs of SUSY, strings/branes and other ingenious proposals. It seems that the simple standard model of particles and cosmology still rules. Nonetheless its plethora of parameters have spurred theorists to search for generalizations of the standard model which may help to cut down the number of arbitrary constants and leave room for mysterious dark matter. Many schemes have been put forward. These usually add other gauge fields, sterile particles, invoke enlarged groups and introduce scalar fields, perhaps associated with cosmological inflation. My feeling is that these ideas are very much hit-or-miss and they do seem to lack a fundamental basis. I think Salam might have looked askance at them. Anyhow here goes...

For many years we have become accustomed to the notion of spacetime events, with local fields (belonging to representations of some gauge group) interacting at a

particular site and time. The $x = (t, \mathbf{x})$ spacetime continuum serves as the backdrop for the ‘when’ and ‘where’ of an event. But, until one specifies the fields involved in the interaction, the ‘what’ of the action is left open, to be determined by experiment. Now we should realise that any event necessarily consists of a transaction or a change of property at a location. (The transaction is usually communicated by a gauge field.) It occurred to me that it might be possible to provide a mathematical backdrop for ‘properties’ or ‘attributes’ of the participating fields by invoking a property space with its own set of coordinates. As far as we can tell there seem to be a finite number of quantum numbers or properties in nature. So the basic idea is to put some mathematics into the ‘what’ of the event by invoking anticommuting (Lorentz scalar) coordinates ζ ; these should serve to provide the setting for the gauge groups and particle attributes and fields should be functions of these ζ as well as spacetime location x . The *full* action is to be integrated over the properties ζ like one does for x . The reason why I have picked ζ as anticommuting is because when an object is endowed by several such properties, the melange is necessarily finite; and since the square of a property vanishes it means that once a fundamental constituent possesses that attribute it cannot doubly have it. Of course since we are dealing with quantum mechanics in the long run, these properties must be complex so the anti-attribute $\bar{\zeta}$ should be permitted. By combining properties with anti-properties one can build up ‘generations’ of particles possessing the same *overall* attributes. In some sense N -extended supersymmetry is based on the same idea but it suffers badly from spin state proliferation.

The question is how many property coordinates ζ are needed? There must be enough to describe the visible world. The pioneers of unified forces^{2,3} have forged the way and provided the inspiration. Despite some criticisms to which these full gauge groups have been subjected, I have opted for SU(5) and SO(10) gauge models; these have many attractive features, so for now I will suppose that there are five independent^a complex ζ . Later on we will be forced to subtly enlarge this number in order to reflect the incontrovertible fact that fermions of distinct chiralities – through their electroweak characteristics – behave quite differently at low energy; thus experiment obliges us to distinguish between left and right properties.

3. Mathematical Description

By enlarging spacetime x with ζ we hope to encompass *all* possible fundamental events. Even though the term has been overused we will assume that there exist ‘superfields’ $\Phi(X)$ and $\Psi(X)$ which are functions of the super-coordinate $X^M \equiv (x^m, \zeta^\mu, \bar{\zeta}^{\bar{\mu}})$. The idea is that an integral over products of just one or two superfields can provide the entire action for every event. The calculus for handling the combination of bosonic x and fermionic ζ is well-established^b and the graded

^aI have found that four ζ are definitely insufficient to produce three generations at least.

^bWe developed this from scratch as we wanted to adhere to Einstein up notation for coordinates and traditional left operations like differentiation. Also we wanted to settle the notation to our

character of X means that Berezinian integration is to be adopted for property integration, with super-determinants coming into play. By curving the superspace we will automatically be able to describe gravity and the other forces of nature, as we shall see.

But let us start with flat space and assume parity conservation; presently we shall improve on this by adding gauge fields and parity violation. With five ζ we are dealing with an overarching $\text{Sp}(10)$ group. The supermetric distance for flat $\text{OSp}(1,3/10)$ is

$$ds^2 = dx^a dx^b \eta_{ba} + \ell^2 (d\zeta^{\bar{\alpha}} d\zeta^{\beta} \eta_{\beta\bar{\alpha}} + d\zeta^{\alpha} d\zeta^{\bar{\beta}} \eta_{\bar{\beta}\alpha})/2, \quad (1)$$

where η_{ba} is Minkowskian and $\eta_{\beta\bar{\alpha}} = -\eta_{\bar{\alpha}\beta} = \delta_{\beta}^{\alpha}$; also a fundamental length scale ℓ must be introduced because we are presuming that property ζ is dimensionless.

The Bose fields are to be associated with even powers of ζ and its conjugate, while the Fermi fields are connected with odd powers. Let us reserve the labels 1,2,3 for colour property or ‘chromicity’ and 0,4 to neutrinicity, electricity. The quantum numbers which are ascribed to these, viz.

$$\text{Charge } Q(\zeta^0, \zeta^{\bar{1}}, \zeta^{\bar{2}}, \zeta^{\bar{3}}, \zeta^4) = (0, 1/3, 1/3, 1/3, -1) \quad (2)$$

$$\text{Fermion Number } F(\zeta^0, \zeta^{\bar{1}}, \zeta^{\bar{2}}, \zeta^{\bar{3}}, \zeta^4) = (1, -1/3, -1/3, -1/3, 1) \quad (3)$$

really only come to life when one introduces the gauge fields, as we soon will. Given the assignments (2), the lepton doublet generations are connected with (ζ^0, ζ^4) , multiplied by powers of $\zeta^{\bar{p}} \zeta^p$; the quark generations arise more subtly.

Component fields ϕ and ψ emerge⁴ when we expand Φ and Ψ as polynomials in ζ & $\bar{\zeta}$. Fermions are to be associated with odd powers and bosons with even powers of attributes. Charge conjugation of course corresponds to the ‘reflection’ operation $\zeta \leftrightarrow \bar{\zeta}$ and we may define a duality operation (that does not affect the $\text{SU}(5)$ representations) under which $(\zeta)^r (\bar{\zeta})^s \leftrightarrow (\zeta)^{5-s} (\bar{\zeta})^{5-r}$. By imposing selfduality *or* anti-selfduality on the superfields we can greatly reduce the number of independent component fields arising in the ζ - expansion This is detailed in ref. 4. In amongst the boson Φ states are nine colour neutral uncharged mesons of which the combination $\zeta^4 \zeta^{\bar{1}} \zeta^{\bar{2}} \zeta^{\bar{3}}$ is recognizable as the standard model Higgs. However, the quark isomultiplets ψ which exist in Ψ are slightly different from the standard model! The up- & down- quarks come as two weak isodoublets/singlets and part of a weak *isotriplet/isodoublet/isosinglet* contained in $\text{SU}(5)$ representations of dimension 45. Thus,

$$\begin{pmatrix} U^{[\bar{\mu}\bar{\nu}]} \sim \zeta^{\bar{\mu}} \zeta^{\bar{\nu}} \zeta^0 \\ D^{[\bar{\mu}\bar{\nu}]} \sim \zeta^{\bar{\mu}} \zeta^{\bar{\nu}} \zeta^4 \end{pmatrix}, \begin{pmatrix} U'^{[\bar{\mu}\bar{\nu}]} \sim \zeta^{\bar{\mu}} \zeta^{\bar{\nu}} \zeta^0 \zeta^{\bar{4}} \zeta^4 \\ D'^{[\bar{\mu}\bar{\nu}]} \sim \zeta^{\bar{\mu}} \zeta^{\bar{\nu}} \zeta^4 \zeta^{\bar{0}} \zeta^0 \end{pmatrix}, \begin{pmatrix} U''^{\lambda} \sim \zeta^{\lambda} \zeta^{\bar{4}} \zeta^0 \\ D''^{\lambda} \sim \zeta^{\lambda} (\zeta^{\bar{0}} \zeta^0, \zeta^{\bar{4}} \zeta^4) \\ X''^{\lambda} \sim \zeta^{\lambda} \zeta^{\bar{0}} \zeta^4 \end{pmatrix}$$

own satisfaction. Hereafter, Latin letters signify spacetime and Greek letters signify property. Early letters of the alphabet connote flatness while later letters imply curved space.

implies the existence of a brand new quark X'' (of charge $-4/3$) in a third generation. Though X'' may be more massive than even the top quark, the consequence at lower energy scales is that we do not expect the CKM matrix to be quite unitary^c. Probably the best way to find X is via a high energy electron-positron collider? Other predictions of the scheme are that heavy leptons should be seen as well as unaccompanied (massive?) D-type quarks. If none of these signals eventuates then it is back to the drawing board and a reexamination to see if any of these ideas about property is salvageable or if the disease is terminal.

4. Force fields

The most interesting feature of our scheme is the way that gauge fields enter and tie in with the quantum number assignments. We note that a flat metric in X is only invariant under global $SU(N)$ unitary rotations of the N attributes. But as soon as we make them local or x dependent, so that

$$\zeta^\mu \rightarrow \zeta'^\mu = [\exp(i\Theta(x))]^{\mu\bar{\nu}} \zeta^\nu \quad (4)$$

we find that there is an inconsistency in the transformation rules for the metric; we are forced to ‘curve’ the space and introduce gauge fields to repair the fault. The way to do this is to write the generalized event (separation)² as

$$ds^2 = dX^M dX^N G_{NM}; \quad G_{NM} = \mathcal{E}_N^B \mathcal{E}_M^A \eta_{AB} (-1)^{[B][M]} \quad (5)$$

where the metric arises through frame vectors \mathcal{E} and the grading is defined in the usual way: $[m] = 0, [\mu] = 1$. Thus the transformations rules for G ,

$$G'_{SR}(X') = \left(\frac{\partial X^M}{\partial X'^R} \right) \left(\frac{\partial X^N}{\partial X'^S} \right) G_{NM}(X) (-1)^{[S]([R]+[M])}, \quad (6)$$

under the local rotations of ζ demand that we introduce components $G_{n\mu}, G_{n\bar{\mu}}$ which have a vectorial character; they should be overall fermionic and must somehow involve the gauge field V as this is the communicator of property across spacetime. A few moment’s reflection (neglecting coupling constants for the present) leads one to the identification $\mathcal{E}_m^\alpha = -iV_m^{\alpha\bar{\nu}} \zeta^\nu$, which is very similar to the way that the em field makes an appearance in the original Klein-Kaluza model; there is really very little room for manoeuvre and the appearance is indeed entirely natural: gauge fields transmit property from one place and time to the next so they ought to arise in the spacetime-attribute sector. The only liberty permitted to us is to multiply by polynomials in property scalars $Z \equiv \zeta^\mu \zeta_\mu$, since these are gauge invariant and carry no quantum numbers. We might say that inclusion of these polynomials corresponds to ‘curving’ property space.

^cThis meshes in with the observation that an isotriplet couples more strongly with the charged W -boson than an isodoublet and therefore the known decay width of the top quark requires a correspondingly smaller V_{tb} coupling to W .

Using that freedom, the only metric which is fully consistent with local $SU(N)$ gauge transformations is

$$\begin{pmatrix} G_{mn} & G_{m\nu} & G_{m\bar{\nu}} \\ G_{\mu n} & G_{\mu\nu} & G_{\mu\bar{\nu}} \\ G_{\bar{\mu}n} & G_{\bar{\mu}\nu} & G_{\bar{\mu}\bar{\nu}} \end{pmatrix} = \begin{pmatrix} g_{mn}C + \ell^2 \bar{\zeta} \{V_m, V_n\} \zeta C'/2 & -i\ell^2 (\bar{\zeta} V_m)^{\bar{\nu}} C'/2 & i\ell^2 (V_m \zeta)^{\nu} C'/2 \\ -i\ell^2 (\bar{\zeta} V_n)^{\bar{\mu}} C'/2 & 0 & \ell^2 \delta_{\mu}^{\nu} C'/2 \\ i\ell^2 (V_n \zeta)^{\mu} C'/2 & -\ell^2 \delta_{\nu}^{\mu} C'/2 & 0 \end{pmatrix}. \quad (7)$$

Here $C(Z) = 1 + \sum_{n=1}^N c_n Z^n$, $C'(Z) = 1 + \sum_{n=1}^N c'_n Z^n$ are independent polynomials of order N in Z which are allowed without destroying the gauge symmetry. One then readily checks that the rule (6) just corresponds to the usual gauge transformation: $iV'_m(x') = \exp[i\Theta(x)](iV_m(x) + \partial_m) \exp[-i\Theta(x)]$. If we just demand subgroup gauge symmetry, we can relax the conditions on the Z polynomials and have them invariant under local subgroup rotations, so more property curvature coefficients c_n can be entertained. We will come back to this when considering QCD plus QED and electroweak theory in such a framework.

The procedure from hereon is pretty straightforward⁵, paying very particular attention to orders of terms and signs that are due to grading. One first constructs the Super-Ricci scalar \mathcal{R} from the Christoffel symbols

$$2\Gamma_{MN}{}^K = [(-1)^{[M][N]} G_{ML,N} + G_{NL,M} - (-1)^{[L]([M]+[N])} G_{MN,L}] (-1)^{[L]} G^{LK} \quad (8)$$

via the Palatini form

$$\mathcal{R} = G^{MK} \mathcal{R}_{KM} = (-1)^{[L]} G^{MK} [(-1)^{[L][M]} \Gamma_{KL}{}^N \Gamma_{NM}{}^L - \Gamma_{KM}{}^N \Gamma_{NL}{}^L]. \quad (9)$$

Secondly one integrates \mathcal{R} over property. This leads to the gravitational and gauge field Lagrangian plus a cosmological term⁶. (As a bonus, the stress tensor T_{mn} of the gauge fields is automatically incorporated in \mathcal{R}_{mn} when we extract the resulting ‘equations of motion’.) The coefficients in front of these terms depends on the number of properties⁷ and on the property curvature coefficients but they all have the generic form

$$\left(\frac{\ell^2}{2}\right)^{2(N-1)} \int d^N \zeta d^N \bar{\zeta} \sqrt{G_{..}} \mathcal{R} = \frac{\mathcal{A} R^{[g]}}{\ell^2} + \mathcal{B} \text{Tr}(F.F) + \frac{\mathcal{C}}{\ell^4}, \quad (10)$$

where $F_{mn} \equiv V_{n,m} - V_{m,n} + i[V_m, V_n]$ and the N -dependent coefficients $\mathcal{A}, \mathcal{B}, \mathcal{C}$ are listed in ref. 7

The matter fields and their Lagrangians are then introduced,

$$\mathcal{L}_{\phi} = \int d^N \zeta d^N \bar{\zeta} \sqrt{-G_{..}} G^{MN} \partial_N \Phi \partial_M \bar{\Phi}; \quad (11)$$

$$\mathcal{L}_{\psi} = \int d^N \zeta d^N \bar{\zeta} \sqrt{-G_{..}} \bar{\Psi} i \Gamma^A E_A{}^M \partial_M \Psi. \quad (12)$$

The gauge and gravitational interactions of the component fields (ϕ, ψ) then just fall out, but these sometimes require wave function renormalizations due to influence of the property curvature coefficients c_n — coefficients which are absent in flat space.

The key point is that the gauge fields couple correctly to the matter fields through the vielbein term

$$E_A{}^M \partial_M \supset e_a{}^m [\partial_m + i(V_m \zeta)^\mu \partial_\mu - i(\bar{\zeta} V_m)^\mu \partial_\mu],$$

so the property derivative is compensated by a further property coordinate attached to the gauge field V ; this is our version of covariant differentiation. Incidentally I ought to declare that such complicated calculations were originally carried using an algebraic computer package devised by Paul Stack and, after time consuming computation, they always produced gauge- and coordinate-invariant results. Knowing this always happened, we have since been able to find a shorter analytic way of picking out the correct terms in (10)-(12) by a procedure which can be generalized to any number of attributes and dispense with Mathematica. Finally, to (11) and (12) we may add the renormalizable super-Yukawa self interactions $\Psi\Phi\Psi$ and $V(\Phi) \simeq \Phi^4$ in the usual manner, with the aim of generating a mass term through the expectation values held in the chargeless fields within $\langle\Phi\rangle$.

Before moving on, three comments about the fermion fields deserve particular mention. Firstly the adjoint field $\bar{\Psi}$ has to be carefully defined with appropriate signs⁸ in property space to produce a series of terms $\bar{\psi}\psi$, after integrating over ζ . Secondly, $\bar{\zeta}\psi$ and their charge conjugates $\psi^{(c)}\zeta$ both appear in the full expansion of $\Psi(\zeta, \bar{\zeta})$ and they simply lead to a doubling of the eventual answers; thus we can simplify calculations by ‘halving’ the expansion of Ψ to $\Psi \supset \zeta\psi$ terms. Thirdly and intriguingly, we have to extend the concept of Dirac γ matrices to super Γ matrices, such that $(\Gamma^A P_A)^2 = \eta^{AB} P_B P_A$. In spacetime we get the standard $\Gamma^a = \gamma^a$ with $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$, but in the property sector one needs to ensure that the ‘square-rooted’ Γ^α are *fermionic* and obey

$$[\Gamma^\alpha, \Gamma^\beta] = [\Gamma^{\bar{\alpha}}, \Gamma^{\bar{\beta}}] = 0, \quad [\Gamma^\alpha, \Gamma^{\bar{\beta}}] = 2\eta^{\alpha\bar{\beta}} = 2\delta_\beta^\alpha. \quad (13)$$

In the same way that Dirac introduced 4×4 matrices and made novel use of the Clifford algebra for spacetime, we must do something similar for property space. We can arrange for the commutators (13) to be satisfied by augmenting property space with auxiliary coordinates θ^α , setting $\Gamma^\alpha \equiv \sigma_+ \theta^\alpha$, $\Gamma^{\bar{\alpha}} \equiv \sigma_- \partial/\partial\theta^\alpha$, and making sure that Ψ is multiplied by the projected singlet $\Theta \equiv (1 + \sigma_3)\theta^1\theta^2 \dots \theta^N/2$, over which one eventually integrates^d. There are probably less extravagant ways of doing this.

5. Electric and Chromic Relativity

To see how all this works out consider QED and QCD which involve one attribute called electricity plus three ‘chromicity’ properties (commonly termed red, green, blue). Thus we confine ourselves to coordinates ζ^1 to ζ^4 and combine both chiralities in Dirac fields since those interactions are blind to parity. Because we are confining ourselves to $U(1) \times SU(3)$ we are dealing with two sets of gauge fields within the fuller

^dOf course the adjoint $\bar{\Psi}$ contains the conjugate singlet $\bar{\Theta} = (1 + \sigma_3)\theta^{\bar{N}} \dots \theta^{\bar{2}}\theta^{\bar{1}}/2$.

SU(4): the em field A and the gluon fields B , having coupling constants e and f respectively. One identifies the frame vectors $\mathcal{E}_m^{\kappa} = -i(fB_m - eA_m/3)^{\kappa\bar{\iota}}\zeta^{\iota}$, $\mathcal{E}_m^4 = ieA_m$, leading to the basic metric elements

$$G_{m4} = i\ell^2\zeta^{\bar{4}}eA_mC'/2, \quad G_{m\iota} = i\ell^2[\zeta^{\bar{\iota}}eA_m/3 - \zeta^{\bar{\kappa}}fB_m^{\kappa\bar{\iota}}]C'/2, \quad (14)$$

which may be multiplied by polynomials in two *distinct* invariants $\zeta^{\bar{\kappa}}\zeta^{\kappa}$ and $\zeta^{\bar{4}}\zeta^4$. I should point out that it is the interactions (14) which actually determine the charge and colour assignments stated in (2) and (3). Also the coupling *must* accompany the gauge fields in order to produce the correct interactions with matter fields.

To simplify the subsequent argument about the resulting interactions I will assume that the property curvature polynomials are common to spacetime & property space:

$$C = C' = 1 + \dots + c_e(\zeta^{\bar{4}}\zeta^4)(\zeta^{\bar{\kappa}}\zeta^{\kappa})^2 + c_f(\zeta^{\bar{\kappa}}\zeta^{\kappa})^3. \quad (15)$$

As we are dealing with four properties, we find that the Berezinian^{9,10} is $\sqrt{-G..} = (2/\ell^2)^4\sqrt{-g..}C^{-2}$. A careful analytical calculation shows that the super-Ricci scalar contains the following gauge field combination:

$$\mathcal{R}\sqrt{-G..} \supset [1 - 3c_e(\zeta^{\bar{4}}\zeta^4)(\zeta^{\bar{\kappa}}\zeta^{\kappa})^2 - 3c_f(\zeta^{\bar{\kappa}}\zeta^{\kappa})^3 + \dots] \sqrt{-g..}g^{km}g^{ln}[4e^2\zeta^{\bar{4}}F_{kl}F_{mn}\zeta^4/3 + f^2\zeta^{\bar{\kappa}}(E_{kl}E_{mn})^{\kappa\bar{\iota}}\zeta^{\iota}], \quad (16)$$

where $F_{mn} = A_{n,m} - A_{m,n}$ and $E_{mn} = B_{n,m} - B_{m,n} + if[B_n, B_m]$ are the standard ‘curls’ of the electromagnetic and gluon fields. The last step is to integrate over the four properties. Including appropriate scaling factors of ℓ^2 one gets

$$\int (d^4\zeta d^4\bar{\zeta}) \mathcal{R}\sqrt{-G..} \supset (-12\sqrt{-g..}/\ell^2)[4c_fe^2F.F + c_ff^2\text{Tr}(E.E)]. \quad (17)$$

Last but not least we must ensure gravitational universality; so we have to set $c_ff^2 = 4c_fe^2$, which is perfectly feasible without demanding equality of the colour and electromagnetic couplings. If we relax the assumption that $C = C'$, it is even easier to ensure universality of Newton’s constant G_N .

The colour and electromagnetic interactions of the matter fields Ψ, Φ emerge from (11) and (12) exactly as expected. See ref. 8. I shall not delve into that because the story is not quite complete and is therefore likely to be misleading: we have neglected neutrinity (the fifth property ζ^0) so the ensuing generations are not the physical ones, as sketched in section 3. To correct for this we must turn to the leptons.

6. Electroweak Relativity

The application of our scheme to the original electroweak model^{11–13} of leptons requires an interesting extension of previous work¹⁴ and leads to an intriguing prediction about the weak mixing angle, not to mention the prediction of two leptonic generations. The fact that the weak isospin and hypercharge assignments of the leptons change with chirality obliges us to invoke distinct properties ζ_L and ζ_R

for left- and right-handed leptons respectively to which the gauge fields latch on (through the frame vectors). The full $SU(4)$ gauge field $V^{\mu\bar{\nu}}$, acting on the pair of doublets $(\zeta_L^0, \zeta_L^4, \zeta_R^0, \zeta_R^4)$ is not needed; only the restricted $SU(2)_L \times U(1)$ rotations demand attention. Thus we reinterpret $V_m = L_m + R_m$, with

$$L_m = (g\mathbf{W}_m \cdot \boldsymbol{\tau} - g'B_m)/2 \quad R_m = g'B_m(\tau_3 - 1)/2, \quad (18)$$

possessing the standard weak hypercharge assignments

$$Y(\zeta_L^0, \zeta_L^4, \zeta_R^0, \zeta_R^4) = (-1, -1, 0, -2). \quad (19)$$

It must also be understood that L is to be associated with the left property derivative $\partial/\partial\zeta_L$ and R is to be associated with the right property derivative $\partial/\partial\zeta_R$; g and g' are the usual coupling constants tied to the weak triplet \mathbf{W} and weak singlet hypercharge B respectively.

It is sufficiently general for our purposes to take the polynomial property curvatures C and C' to be equal and direct products of quadratic left- and right-handed polynomials:

$$C = C_R C_L = [1 + c_R Z_R + c_{RR} Z_R^2][1 + c_L + c_{LL} Z_L^2]; \quad Z_R \equiv \bar{\zeta}_R \zeta_R, \quad Z_L \equiv \bar{\zeta}_L \zeta_L. \quad (20)$$

These enter in the metric components:

$$G_{m\zeta_L} = -i\ell^2 \bar{\zeta}_L L_m C/2; \quad G_{m\zeta_R} = -i\ell^2 \bar{\zeta}_R R_m C/2, \quad (21)$$

$$G_{\zeta_L \bar{\zeta}_L} = G_{\zeta_R \bar{\zeta}_R} = \ell^2 C/2, \quad G_{\zeta_L \zeta_R} = G_{\bar{\zeta}_L \bar{\zeta}_R} = G_{\zeta_L \bar{\zeta}_R} = G_{\zeta_R \bar{\zeta}_L} = 0. \quad (22)$$

The remaining metric element reads

$$G_{mn} = C[g_{mn} + (\text{gauge field terms})]. \quad (23)$$

Factorisability of C simplifies the calculations enormously when we integrate over the whole eight properties: $\int d^2\zeta_R d^2\bar{\zeta}_R d^2\zeta_L d^2\bar{\zeta}_L$.

The various contributions to the super-Ricci scalar drop out as follows, bearing in mind that $\sqrt{-G..} = (2/\ell^2)^4 \sqrt{-g..} (C_R C_L)^{-3}$. There are three terms:

$$\int d^2\zeta_R.. d^2\bar{\zeta}_L \sqrt{-G..} \mathcal{R} \supset 36\sqrt{-g..} (2/\ell^2)^4 R^{[g]} (2c_R^2 - c_{RR})(2c_L^2 - c_{LL}), \quad (24)$$

$$\begin{aligned} \int d^2\zeta_R.. d^2\bar{\zeta}_L \sqrt{G..} \mathcal{R} \supset & -\frac{3}{2}\sqrt{g..} \left(\frac{2}{\ell^2}\right)^3 [c_L(3c_R^2 - 2c_{RR})(g^2 \mathbf{W}_{mn} \cdot \mathbf{W}^{mn} + g'^2 B_{mn} B^{mn}) \\ & + g'^2 2c_R(3c_L^2 - 2c_{LL})B_{mn} B^{mn}], \end{aligned} \quad (25)$$

$$\int d^2\zeta_R.. d^2\bar{\zeta}_L \sqrt{G..} \mathcal{R} \supset 12\sqrt{g..} (2/\ell^2)^5 [(4c_L c_{LL} - 5c_L^2)(2c_R^2 - c_{RR}) + (L \leftrightarrow R)], \quad (26)$$

where $W_{mn} \equiv W_{n,m} - W_{m,n} + ig[W_n, W_m]$ and $B_{mn} \equiv B_{n,m} - B_{m,n}$. The full answer is the sum of (24)-(26).

Universality of gravity at the semiclassical level anyway (and the correct normalization of the gauge fields) is guaranteed when we set

$$c_L(3c_R^2 - 2c_{RR})(g^2 - g'^2) = 2c_R(3c_L^2 - 2c_{LL})g'^2,$$

which is readily arranged. But much more intriguing is the fact that if the property curvature is parity conserving so that $c_R = c_L = c$, $c_{RR} = c_{LL} = c_2$ and implying that all parity violation comes from the gauge fields in the frame vectors, then $g^2 = 3g'^2$. Thus the weak angle reduces to 30° . It makes good sense because the property curvature C polynomial accompanies the gravitational field and, as far as we know, gravity does not know the left hand from the right. So this restriction seems very natural and the value of the weak angle is a consequence of gravitational universality in this framework; it is not a result of invoking a higher group or anomaly cancellation, as some other analyses^{15,16} would have.

Turning to the matter fields, we can reduce the number of components by invoking selfduality^e (corresponding to symmetry about the cross diagonal in the superfield expansions). Ignoring the charge conjugate terms, which simply double the results below, two fermion generations, ψ and ψ' arise from expanding Ψ . Using the shorthand symbols $Z_L \equiv \bar{\zeta}_L \zeta_L$, $Z_R \equiv \bar{\zeta}_R \zeta_R$ as in (20), we get

$$2\Psi = \bar{\zeta}_L[\psi_L(1 + Z_R^2/2) + \psi'_L Z_R](1 + Z_L) + (L \leftrightarrow R), \quad (27)$$

$$2\bar{\Psi} = [\bar{\psi}_L(1 + Z_R^2/2) + \bar{\psi}'_L Z_R]\zeta_L(1 + Z_L) + (L \leftrightarrow R). \quad (28)$$

Since chirality ensures that $\bar{\psi}_L \psi_L = \bar{\psi}_R \psi_R = 0$, we find that a mass term arising from the product $\bar{\Psi}\Psi$ has insufficient powers of ζ to give a nonzero answer; thus a *mass term vanishes identically* and this is a good thing because it indicates that we need to couple fermions to bosons before one can generate mass. The kinetic term is fine however; in flat space,

$$- \int d^2\zeta_R d^2\bar{\zeta}_L \bar{\Psi} i\gamma \cdot \partial \Psi = \bar{\psi}_L i\gamma \cdot \partial \psi_L + \bar{\psi}'_L i\gamma \cdot \partial \psi'_L + (L \leftrightarrow R). \quad (29)$$

Regarding the bosons, we recall that the selfdual combinations are $(1 + Z^2/2)$ and Z with $\zeta\bar{\zeta} \rightarrow 0$, separately for left- and right-handed properties. Hence the fully selfdual, hermitian superBose field Φ is

$$2\Phi = \varphi(1 + Z_L^2/2)(1 + Z_R^2/2) + \varphi' Z_L Z_R + \Lambda Z_L(1 + Z_R^2/2) + P Z_R(1 + Z_L^2/2) + [\bar{\zeta}_R \phi \zeta_L + \bar{\zeta}_L \phi^\dagger \zeta_R + \phi' \zeta_R \zeta_L + \bar{\zeta}_L \bar{\zeta}_R \phi'^\dagger](1 + Z_L)(1 + Z_R). \quad (30)$$

If we further restrict ourselves to fields of even parity under the operation $\zeta_R \leftrightarrow \zeta_L$, we find $\Lambda = P \equiv \chi/\sqrt{2}$, $\varphi' = 0$, $\phi = \phi'$, so the expansion (30) reduces to

$$2\Phi = \varphi(1 + Z_L^2/2)(1 + Z_R^2/2) + \chi[Z_L(1 + Z_R^2/2) + Z_R(1 + Z_L^2/2)]/\sqrt{2} + [\bar{\zeta}_R \phi \zeta_L + \bar{\zeta}_L \phi^\dagger \zeta_R](1 + Z_L)(1 + Z_R). \quad (31)$$

^eSU(2) duality, indicated by \times , stipulates that $1^\times = Z^2/2$, $(\zeta^\mu)^\times = (\zeta^\mu)Z$, $Z^\times = Z$, $(\eta_{\mu\nu}\zeta^\mu\zeta^\nu)^\times = -\eta_{\mu\nu}\zeta^\mu\zeta^\nu$, where $Z = \zeta^\mu\bar{\zeta}_\mu$. Vice versa, and likewise for the hermitian conjugate combinations. Thus the selfdual combinations are $(1 + Z^2/2)$, Z and $\zeta(1 + Z)$ with $\eta_{\mu\nu}\zeta^\mu\zeta^\nu \rightarrow 0$. We apply this separately to left and right leptonic properties in the following equations, corresponding to the subgroup $SU(2)_L \times SU(2)_R$.

The normalization factors have been concocted so that

$$- \int d^2 \zeta_R \dots d^2 \bar{\zeta}_L \Phi^2 = -\varphi^2 - \varphi'^2 - \chi^2 + \text{Tr}(\phi^2). \quad (32)$$

In (31) the quartet $\phi^{\mu\bar{\nu}} = (\phi_o I + \phi.\tau)^{\mu\bar{\nu}}/\sqrt{2}$ consists of a singlet and a triplet. The quantum numbers $I_{3L}, Y, Q = I_{3L} + Y/2$ of the components read:

$$\begin{aligned} Y(\varphi, \varphi', \chi) &= (0, 0, 0); & I_{3L}(\varphi, \varphi', \chi) &= (0, 0, 0); & Q(\varphi, \varphi', \chi) &= (0, 0, 0); \\ Y(\phi^{00}, \phi^{04}, \phi^{40}, \phi^{44}) &= (1, 1, -1, -1); & 2I_{3L}(\phi^{00}, \phi^{04}, \phi^{40}, \phi^{44}) &= (-1, 1, -1, 1); \\ Q(\phi^{00}, \phi^{04}, \phi^{40}, \phi^{44}) &= (0, 1, -1, 0). \end{aligned}$$

The Higgs boson will be associated with $\phi_0 + \phi_3$, as we will presently discover. For that identification we need to consider the super-Yukawa and gauge field interactions in flat spacetime, before we curve spacetime with gravity.

With L and R gauge fields defined in (18), the vielbeins which correspond to the metric elements (21) - (23) are:

$$\begin{pmatrix} E_a^m & E_a^\mu & E_a^{\bar{\mu}} \\ E_\alpha^m & E_\alpha^\mu & E_\alpha^{\bar{\mu}} \\ E_{\bar{\alpha}}^m & E_{\bar{\alpha}}^\mu & E_{\bar{\alpha}}^{\bar{\mu}} \end{pmatrix} = \frac{1}{\sqrt{C}} \begin{pmatrix} e_a^m i[(L_a \zeta_L) + (R_a \zeta_R)]^\mu - i[(\bar{\zeta}_L L_a) + (\bar{\zeta}_R R_a)]^{\bar{\mu}} \\ 0 & \delta_\alpha^\mu & 0 \\ 0 & 0 & \delta_{\bar{\alpha}}^{\bar{\mu}} \end{pmatrix}. \quad (33)$$

Thus the fermion kinetic energy can be written in the form $\bar{\Psi} i \Gamma^A D_A \Psi$, where

$$D_A = E_A^M \partial_M = E_A^m \partial_m + E_A^\mu \partial_\mu + E_A^{\bar{\mu}} \partial_{\bar{\mu}}$$

acts like a covariant derivative. Let V serve as a generic gauge field; the action of $i\gamma^a D_a$ on $f(Z)(\bar{\zeta}\psi)$ is to give $f(Z)\bar{\zeta}\gamma.(i\partial_a + V_a)\psi$ and on $f(Z)(\bar{\psi}\zeta)$ is to give $f(Z)(i\partial_a + V_a)\bar{\psi}\gamma^a\zeta$. So when we integrate over property we end up precisely with the usual gauge field interaction $\bar{\psi}\gamma^a(i\partial_a + V_a)\psi$ for each of the two generations, which in the leptonic case translates into

$$\bar{\psi}_L \gamma^a (i\partial_a + L_a) \psi_L + \bar{\psi}_R \gamma^a (i\partial_a + R_a) \psi_R + (\psi \rightarrow \psi').$$

This is unsurprising; interpreting $(\psi^0, \psi^4) = (\nu, l)$, one ends up with the standard

$$\begin{aligned} \mathcal{L}_\psi &= \bar{l}\gamma.(i\partial - eA)l + \bar{\nu}i\gamma.\partial\nu + \frac{e}{\sqrt{2}\sin\theta} [\bar{\nu}_L \gamma.W^+ l_L + \bar{l}_L \gamma.W^- \nu_L] \\ &+ \frac{e}{\sin 2\theta} (\bar{\nu}_L \gamma.Z \nu_L) + e \tan\theta (\bar{l}_R \gamma.Z l_R) - e \cot 2\theta (\bar{l}_L \gamma.Z l_L) \\ &+ (l, \nu) \rightarrow (l', \nu'), \end{aligned} \quad (34)$$

where $\cos\theta = g/\sqrt{g^2 + g'^2}$, $\sin\theta = g'/\sqrt{g^2 + g'^2}$, $e = gg'/\sqrt{g^2 + g'^2}$. (34) simplifies to a considerable extent when $\theta = 30^\circ$, as indicated by gravitational universality, because the Z field then interacts purely axially with the charged lepton, in contrast to the purely vectorial electromagnetic field.

But when we come to the bosons we discover something new. Acting with the covariant derivative on the Bose superfield,

$$D_a \Phi = [E_a^m \partial_m + E_a^\mu \partial_\mu + E_a^{\bar{\mu}} \partial_{\bar{\mu}}] \Phi = [\partial_a + i(V_a \zeta)^\mu \partial_\mu - i(\bar{\zeta} V_a)^{\bar{\mu}} \partial_{\bar{\mu}}] \Phi.$$

Referring to eq. (31) we obtain

$$2D\Phi.D\Phi = (1+2Z_L)(1+2Z_R)[\bar{\zeta}_R\{\partial\phi+i(\phi L-R\phi)\}\zeta_L\bar{\zeta}_L\{\partial\phi+i(\phi R-L\phi)\zeta_R\}] \quad (35)$$

plus terms which disappear when integrated over property. If we concentrate on the uncharged fields held in the quartet ϕ , viz. ϕ^{00} & ϕ^{44} , that occur on the diagonal (or equivalently ϕ_0 & ϕ_3), we find that

$$2\text{Tr}[(\phi R - L\phi)(\phi L - R\phi)] \rightarrow \frac{1}{2}g^2W^+W^-(\phi_+^2 + \phi_-^2) + \frac{1}{4}\phi_+^2(gW_3 - g'B)^2 + \frac{1}{4}\phi_-^2(gW_3 + g'B)^2 - g'^2\phi_-^2; \quad \phi_{\pm} \equiv \phi_0 \pm \phi_3.$$

In order to recover the standard vector meson masses, we must therefore take

$$\langle\phi_-\rangle = 0 \text{ or } \langle\phi_0\rangle = \langle\phi_3\rangle, \text{ and } \langle\phi_+\rangle = v,$$

for the expectation values, whereupon

$$\begin{aligned} \langle 2\text{Tr}[(\phi R - L\phi)(\phi L - R\phi)] \rangle &\rightarrow \frac{1}{2}v^2g^2W^+W^- + \frac{1}{4}v^2(g^2 + g'^2)Z^2 \\ &= \frac{e^2v^2}{2\sin^2\theta}W^+W^- + \frac{e^2v^2}{\sin^2 2\theta}Z^2. \end{aligned} \quad (36)$$

All is as it should be and the em field A remains massless.

Given these expectation values, we turn to the Yukawa interaction of the super-Bose field Φ with the super-Fermion field Ψ . Before launching into this we need to remind ourselves that in order to get masses for leptons *as well as neutrinos*, we have to consider the Higgs doublet H as well as its doublet counterpart $i\tau_2 H^*$. In our context it means that we have to consider ϕ as well $\tau_2\phi^*\tau_2$. Since we will be integrating over the ζ and the fermion pieces involve $\bar{\zeta}_R\zeta_L$ or $\bar{\zeta}_L\zeta_R$, we need to pick out matching bose pieces. Using the acceptable combination^f $\hat{\phi} = c_l\tau_2\phi^*\tau_2 + s_l\phi$, in place of ϕ , we then find that

$$\begin{aligned} -8\sqrt{2}\bar{\Psi}\Phi\Psi &\supset (\bar{\zeta}_R\hat{\phi}\zeta_L + \bar{\zeta}_L\hat{\phi}^\dagger\zeta_R)(1+2Z_L)(1+2Z_R). \\ &[\bar{\zeta}_L\zeta_R(\bar{\psi}_R + Z_R\bar{\psi}'_R)(\psi_L + Z_L\psi'_L) + (R \leftrightarrow L)]. \end{aligned}$$

Consequently, integrating over property produces a mixture of the two generations:

$$-16 \int d^2\zeta_R..d^2\bar{\zeta}_L \bar{\Psi}\Phi\Psi = (2\bar{\psi} + \bar{\psi}')\hat{\phi}(2\psi + \psi') \equiv 5\bar{\psi}\hat{\phi}\psi.$$

Taking expectation values of Φ to generate a fermionic mass term, and recalling that $\langle\phi_-\rangle = 0$, the Yukawa term (including a coupling constant \mathbf{g}) reduces to

$$5\mathbf{g}(\bar{\nu}_l, \bar{l}) \begin{pmatrix} c_l\langle\phi_+\rangle & 0 \\ 0 & s_l\langle\phi_+\rangle \end{pmatrix} \begin{pmatrix} \nu_l \\ l \end{pmatrix} = 5v\mathbf{g}(c_l\bar{\nu}_l\nu_l + s_l\bar{l}l). \quad (37)$$

^fWith such a combination, $\text{Tr } \hat{\phi}^2 = (c_l^2 + s_l^2)(\phi_0^2 + \phi^2) + 2c_ls_l(\phi_0^2 - \phi^2)$. So taking expectation values, $\text{Tr } \langle\hat{\phi}\rangle^2 = 2(c_l^2 + s_l^2)v^2 \rightarrow 2v^2$ if we interpret $c_l \equiv \cos\theta_l$, $s_l \equiv \sin\theta_l$.

The other mixture $\tilde{\psi} = (-\psi + 2\psi')/\sqrt{5}$ does not acquire a mass in this model. If we were to stretch credulity and pretend we have a decent model for leptons we would be inclined to associate $\hat{\psi}$ with the muonic doublet and $\tilde{\psi}$ with the electronic one; but all this is academic: we really need the three colour properties to corral the known leptonic generations.

The last thing to consider is the effect of spacetime curvature (through e_m^a or g_{mn}) and of property curvature $C(Z)$ on the above results. The effect of e is very simple: it just serves to make the interactions generally covariant and we have nothing more to add to that. The effect of C enters through the Berezinian

$$\sqrt{G_{..}} = \sqrt{g_{..}}(2/\ell^2)^4 C^{-2} \propto (1 - 2c_R - 2c_{RR}Z_{RR} + 3c_R^2 Z_R^2)(1 - 2c_L - 2c_{LL}Z_{LL} + 3c_L^2 Z_L^2).$$

It is subtler and causes mixing as well as wavefunction renormalization. To see what happens, consider the kinetic term of the fermions and simplify the argument by assuming the property curvature is blind to parity as we did before to recover a weak mixing angle of 30° . In that case, using the expanded

$$\begin{aligned} \sqrt{G_{..}} = \sqrt{g_{..}}(2/\ell^2)^4 [1 - 2c(Z_R + Z_L) + 4c^2 Z_R Z_L + \\ (3c^2 - 2c_2)\{Z_R^2(1 - 2cZ_L) + Z_L^2(1 - 2cZ_R)\} + \{(3c^2 - 2c_2)Z_L Z_R\}^2] \end{aligned} \quad (38)$$

. we obtain, after ζ integration, the kinetic term ($D \equiv \partial - iV$),

$$\sqrt{g_{..}}[(1 - c)\{\bar{\psi}i\gamma.D\psi + \bar{\psi}'i\gamma.D\psi' - 2c(\bar{\psi}i\gamma.D\psi' + \bar{\psi}'i\gamma.D\psi)\} + c(c^2 + 2c_2)\bar{\psi}i\gamma.D\psi]$$

which reduces to (29) when $c \rightarrow 0$. Thus the curving of property engenders source field mixing and wavefunction renormalization, without affecting the coupling of the gauge field V configuration. Similar conclusions apply to the Bose sector.

7. Generalizations and Conclusions

I have outlined the main consequences of a mathematical scheme for handling the ‘when-where-what’ of events by an enlarged coordinate backdrop, part being commuting (spacetime) and part anticommuting (property). It automatically produces a finite number of generations of elementary particles and provides a framework that unifies gravity with the other forces of nature. We treated the case of strong and electromagnetic interactions $SU(3) \times U(1)$ corresponding to three chromicity and one electricity property, making for a total of four P-conserving properties. Then we considered augmenting these by neutrinity to describe electroweak theory and there we found the need to distinguish between left and right leptons. Thus the minimal number of properties ζ for encompassing the known forces is 5 (or 7 if we double up for leptonic handedness). The full story requires the use of them all and I admit to not having properly tackled that yet. It is a daunting business as you have seen from the calculations presented earlier. We went on to show that if the property curvature coefficients respect parity – which befits gravity at any rate – the weak mixing angle must equal 30° to guarantee gravitational universality. Also we proved that the simplest generalization of the standard electroweak model resulted

in *two* lepton generations, one massive and one massless, and in addition was able to reproduce what we know about vector masses. We remain nonplussed as to how to constrain the coefficients c_n which curve property and we are still searching for a principle that will do the job.

To fully handle the complete $SU(3) \times SU(2)_L \times U(1)$ gauge group, rather than bits and pieces, will require more calculational acrobatics and is left for future research. Suffice it to say that we have come across obstacles and have so far circumvented them all. Whether we will be able to overcome looming problems is quite another matter: it may well turn out that the predictions which emerge will not be able to withstand experimental scrutiny. We have set our sights on reproducing the standard model, with the particle generations automatically catered for. If this succeeds, one can look farther afield, seeing as we have barely scratched the surface of the scheme. A left-right symmetric picture beckons; sterile states that do not interact with the basic constituents exist aplenty in the expansions of Ψ and Φ and, if we think fancifully, may have connections to dark matter; finally the quantization via BRST seems to find a natural place in our framework since it introduces anticommuting scalar variables attached to the ghost fields, leading to an $Sp(2)$ translation group. On a more cautionary note, the future may judge the entire approach as being completely misguided; after all, just one ugly fact can slay a beautiful hypothesis. The history of physics is littered with such failures. If so, the present scheme can be buried with lots of other valiant attempts in the graveyard of failed theories, but its ghost may linger awhile.

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